SCORE: 19 / 20 POINTS

1. **NO CALCULATORS ALLOWED**
2. **UNLESS STATED OTHERWISE, YOU MUST SIMPLIFY ALL ANSWERS**
3. **SHOW PROPER & CONCISE PRECALCULUS LEVEL WORK TO JUSTIFY YOUR ANSWERS**

Using complete sentences, write the distance based definition of "ellipse".   SCORE: 2 / 2 PTS

An ellipse is the locus of points on a plane (x, y) whose distances to two fixed points add up to a fixed constant (the two fixed points are called foci).
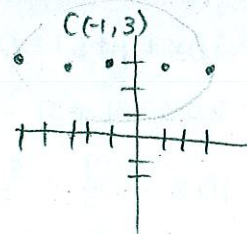

Find the foci and vertices of the ellipse $3x^2 + 4y^2 + 6x - 24y - 9 = 0$.   SCORE: 5 / 5 PTS

$$3x^2 + 6x + 4y^2 - 24y = 9 \quad \left(\tfrac{1}{2}\right)$$

$$\left(\tfrac{1}{2}\right) \; 3(x^2 + 2x + 1) + 4(y^2 - 6y + 9) = 9 + 3 + 36 \quad \left(\tfrac{1}{2}\right)$$

$$\frac{3(x+1)^2}{48} + \frac{4(y-3)^2}{48} = \frac{48}{48} \quad \left(\tfrac{1}{2}\right)$$

$$\frac{(x+1)^2}{16} + \frac{(y-3)^2}{12} = 1 \quad \left(\tfrac{1}{2}\right)$$

center: $(-1, 3)$ $\left(\tfrac{1}{2}\right)$
vertices: $(-5, 3), (3, 3)$ ①
foci: $(-3, 3), (1, 3)$ ①

$$\begin{array}{r} 16 \\ 3\,\overline{)48} \\ \underline{3} \\ 18 \end{array}$$

$$16 = 12 + c^2$$
$$4 = c^2$$
$$2 = c$$

$C(-1, 3)$

The focus of a parabola is at the origin, and its vertex is at $(3, 0)$.   SCORE: 2 / 2 PTS
Find the equation of the parabola.

$(3, 0)$
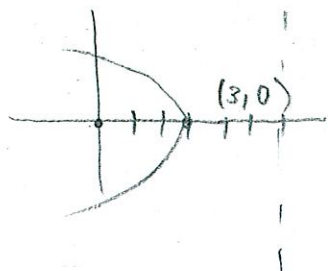
$$y^2 = 4px$$
$$y^2 = 4(-3)x$$
$$y^2 = -12x$$

$$\boxed{y^2 = -12(x-3)}$$
$\left(\tfrac{1}{2}\right)$ ① $\left(\tfrac{1}{2}\right)$

Fill in the blanks. **NO NEED TO SHOW WORK.** ①   SCORE: **2** / 3 PTS

[a]   The latera recta of an ellipse are perpendicular to **the axis** and pass through **the foci**.

[b]   The eccentricity of the ellipse $\dfrac{x^2}{12} + \dfrac{y^2}{13} = 1$ is $\boxed{\dfrac{\sqrt{13}}{13}}$ ①

$e = \dfrac{c}{a}$

$13 = 12 + c^2$
$1 = c^2$
$\sqrt{1} = c$

$\dfrac{c}{a} = \dfrac{\sqrt{1}}{\sqrt{13}} \cdot \dfrac{\sqrt{13}}{\sqrt{13}} = \dfrac{\sqrt{13}}{13}$

Find the equation of the ellipse with foci $(-7, 5)$ and $(-7, -3)$, and a major axis of length 10.   SCORE: **4** / 4 PTS

center : $(-7, 1)$ ①/₂

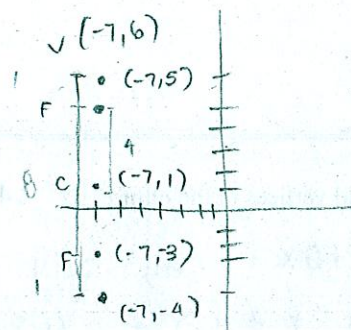$\dfrac{(x+7)^2}{m^2} + \dfrac{(y-1)^2}{25} = 1$

$a = 5$
$c = 4$

$25 = m^2 + 16$
①/₂ $9 = m^2$

$$\boxed{\dfrac{(x+7)^2}{9} + \dfrac{(y-1)^2}{25} = 1}$$
① ①/₂ ① ①/₂

✓ $(-7, 6)$
$(-7, 5)$
F
4
$(-7, 1)$
C
F $(-7, -3)$
$(-7, -4)$

Find the vertex, focus and equation of the directrix of the parabola $2x^2 - 20x + y + 51 = 0$.   SCORE: **4** / 4 PTS

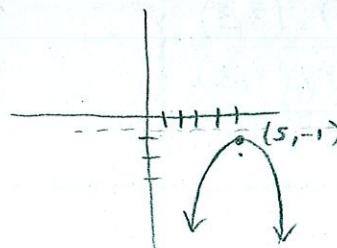$2x^2 - 20x + y + 51 = 0$

$\dfrac{1}{2}(2x^2 - 20x + y + 51 = 0)$

$x^2 - 10x + \dfrac{y}{2} + \dfrac{51}{2} = 0$

①/₂ $x^2 - 10x + 25 = -\dfrac{y}{2} - \dfrac{51}{2} + \dfrac{50}{2}$ ①/₂

$(x-5)^2 = -\dfrac{y}{2} - \dfrac{1}{2}$ ①/₂

$(x-5)^2 = -\dfrac{1}{2}(y+1)$ ①/₂

$\boxed{\text{vertex : } (5, -1)}$ ①/₂

$4p = -\dfrac{1}{2}$

①/₂ $p = -\dfrac{1}{8}$

$(5, -1 - \dfrac{1}{8})$

$\boxed{\text{focus : } (5, -\dfrac{9}{8})}$ ①/₂

$(5, -1)$

$\boxed{\text{directrix : } y = -\dfrac{7}{8}}$ ①/₂

$-\dfrac{8}{8} + \dfrac{1}{8} = -\dfrac{7}{8}$